Embrace Coaching Academy
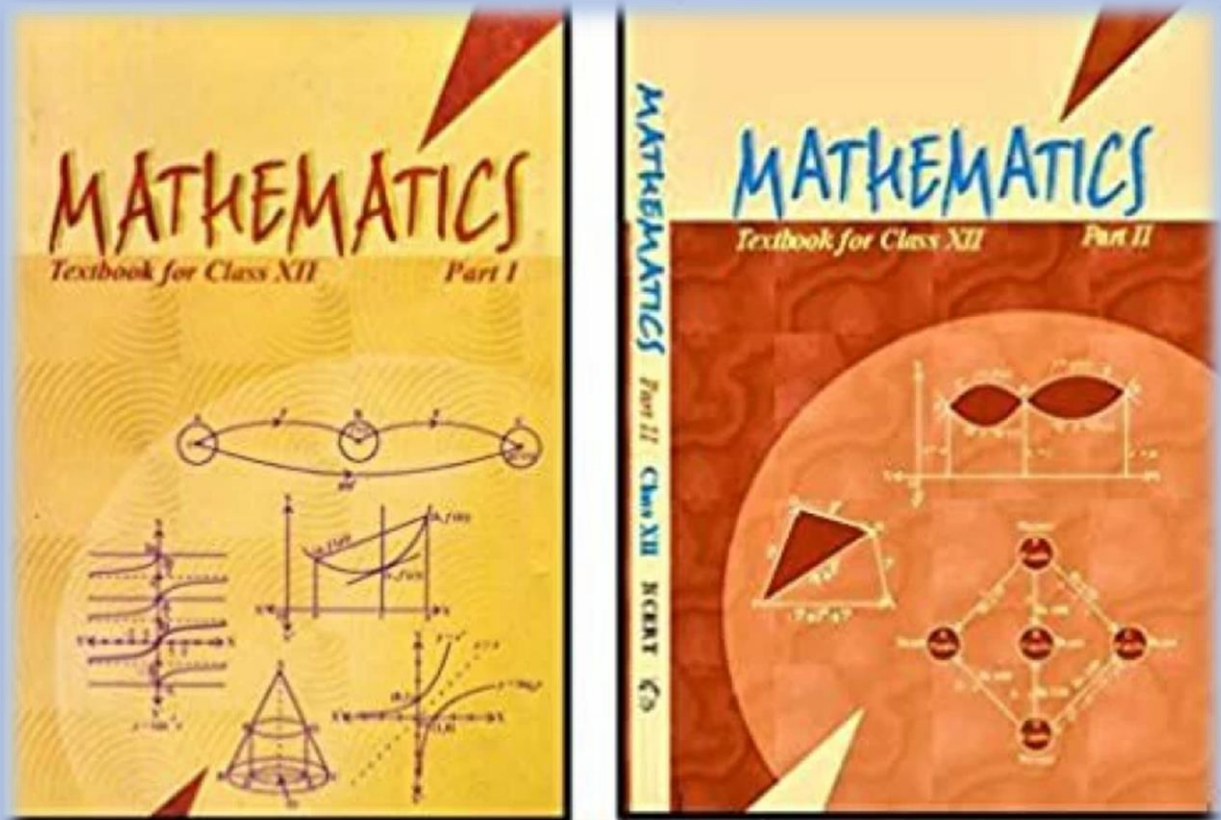
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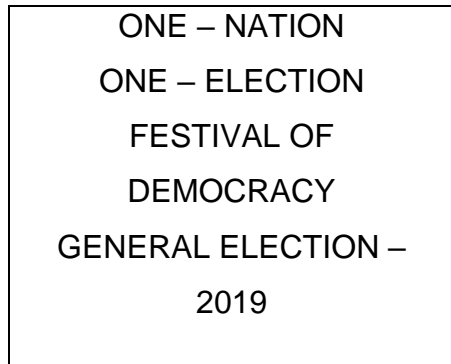

CBSE Class 12 – Mathematics

Case studies

Relation and Function

CASE STUDY 1:

A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever

Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation 'R' is defined on I as follows:

R = {$(V1, V2)$: $V1, V2$ ∈ I and both use their voting right in general election – 2019}

1. Two neighbors X and Y∈ I. X exercised his voting right while Y did not cast her vote in general election – 2019. Which of the following is true?
a. (X,Y) ∈R
b. (Y,X) ∈R
c. (X,X) ∉R
d. (X,Y) ∉R

2. Mr.'X' and his wife 'W'both exercised their voting right in general election -2019, Which of the following is true?
a. both (X,W) and (W,X) ∈ R
b. (X,W) ∈ R but (W,X) ∉ R
c. both (X,W) and (W,X) ∉ R
d. (W,X) ∈ R but (X,W) ∉ R

3. Three friends F₁, F2 and F₃ exercised their voting right in general election-2019, then which of the following is true?
a. (F1,F2) ∈R, (F2,F3) ∈ R and (F1,F3) ∈ R
b. (F1,F2) ∈ R, (F2,F3) ∈ R and (F1,F3) ∉ R
c. (F1,F2) ∈ R, (F2,F2) ∈R but (F3,F3) ∉ R
d. (F1,F2) ∉ R, (F2,F3) ∉ R and (F1,F3) ∉ R

4. The above defined relation R is _____

a. Symmetric and transitive but not reflexive

b. Universal relation

c. Equivalence relation

d. Reflexive but not symmetric and transitive

5. Mr. Shyam exercised his voting right in General Election – 2019, then Mr. Shyam is related to which of the following?

a. All those eligible voters who cast their votes

b. Family members of Mr.Shyam

c. All citizens of India

d. Eligible voters of India

ANSWERS

1. (d) (X,Y) $\notin$ R

2. (a) both (X,W) and (W,X) $\in$ R

3. (a) (F1,F2) $\in$ R, (F2,F3) $\in$ R and (F1,F3) $\in$ R

4. (c) Equivalence relation

5. (a) All those eligible voters who cast their votes

CASE STUDY 2

Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set {1,2,3,4,5,6}. Let A be the set of players while B be the set of all possible outcomes.

A = {S, D}, B = {1,2,3,4,5,6}

1. Let $R : B \rightarrow B$ be defined by R = $\{(x,y): y \text{ is divisible by } x \}$ is
a. Reflexive and transitive but not symmetric
b. Reflexive and symmetric and not transitive
c. Not reflexive but symmetric and transitive
d. Equivalence

2. Raji wants to know the number of functions from A to B. How many number of functions are possible?
a. 6^2
b. 2^6
c. 6!
d. 2^{12}

3. Let R be a relation on B defined by R = {(1,2), (2,2), (1,3), (3,4), (3,1), (4,3), (5,5)}. Then R is
a. Symmetric
b. Reflexive
c. Transitive
d. None of these three

4. Raji wants to know the number of relations possible from A to B. How many numbers of relations are possible?
    a. 6^2
    b. 2^6
    c. 6!
    d. 2^{12}

5. Let $R: B \rightarrow B$ be defined by R={(1,1),(1,2), (2,2), (3,3), (4,4), (5,5),(6,6)}, then R is
a. Symmetric
b. Reflexive and Transitive
c. Transitive and symmetric
d. Equivalence

ANSWERS

1. (a) Reflexive and transitive but not symmetric
2. (a) 6^2
3. (d) None of these three
4. (d) 2^{12}
5. (b) Reflexive and Transitive

CASE STUDY 3:

An organization conducted bike race under 2 different categories-boys and girls. Totally there were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let B = {b$_1$,b$_2$,b$_3$} G={g$_1$,g$_2$} where B represents the set of boys selected and G the set of girls who were selected for the final race.

Ravi decides to explore these sets for various types of relations and functions

1. Ravi wishes to form all the relations possible from B to G. How many such relations are possible?
   a. 2^6
   b. 2^5
   c. 0
   d. 2^3

2. Let R: B→B be defined by R = {(x, y): x and y are students of same sex}, Then this relation R is_____
   a. Equivalence
   b. Reflexive only
   c. Reflexive and symmetric but not transitive
   d. Reflexive and transitive but not symmetric

3. Ravi wants to know among those relations, how many functions can be formed from B to G?
   a. 2^2
   b. 2^{12}
   c. 3^2
   d. 2^3

4. Let $R: B \rightarrow G$ be defined by R = { (b$_1$,g$_1$), (b$_2$,g$_2$),(b$_3$,g$_1$)}, then R is_____

a. Injective

b. Surjective

c. Neither Surjective nor Injective

d. Surjective and Injective

5. Ravi wants to find the number of injective functions from B to G. How many numbers of injective functions are possible?

a. 0

b. 2!

c. 3!

d. 0!

ANSWERS

1. (a) 2^6

2. (a) Equivalence

3. (d) 2^3

4. (b) Surjective

5. (a) 0

CASE STUDY 5:

Students of Grade 9, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line $y = x - 4$. Let L be the set of all lines which are parallel on the ground and R be a relation on L.

Answer the following using the above information.

1. Let relation R be defined by R = $\{(L1, L2): L1 \parallel L2$ where $L_1, L_2 \in L\}$ then R is_____ relation

a. Equivalence

b. Only reflexive

c. Not reflexive

d. Symmetric but not transitive

2. Let R = { $(L1, L2)$: $L1 \perp L2$ where L1, L2 $\in$ L } which of the following is true?

a. R is Symmetric but neither reflexive nor transitive

b. R is Reflexive and transitive but not symmetric

c. R is Reflexive but neither symmetric nor transitive

d. R is an Equivalence relation

3. The function f: R→R defined by $f(x) = x - 4$ is_____

a. Bijective

b. Surjective but not injective

c. Injective but not Surjective

d. Neither Surjective nor Injective

4. Let $f: R \to R$ be defined by $f(x) = x - 4$. Then the range of $f(x)$ is _____

a. R

b. Z

c. W

d. Q

5. Let R = {(L$_1$, L$_2$) : L$_1$ is parallel to L$_2$ and L$_1$: y = x − 4} then which of the following can be taken as L$_2$?

a. 2x-2y+5=0

b. 2x+y=5

c. 2x + 2y + 7 =0

d. x+y=7

ANSWERS

1. (a) Equivalence

2. (a) R is Symmetric but neither reflexive nor transitive

3. (a) Bijective

4. (a) R

5. (a) 2x -2y +5 =0

CASE STUDY 5:

Raji visited the Exhibition along with her family. The Exhibition had a huge swing, which attracted many children. Raji found that the swing traced the path of a Parabola as given by $y = x^2$.

Answer the following questions using the above information.

1. Let $f: R \rightarrow R$ be defined by $f(x) = x^2$ is_____
   a. Neither Surjective nor Injective
   b. Surjective
   c. Injective
   d. Bijective

2. Let $f: N \rightarrow N$ be defined by $f(x) = x^2$ is _____
   a. Surjective but not Injective
   b. Surjective
   c. Injective
   d. Bijective

3. Let f: {1,2,3,....}→{1,4,9,....} be defined by $f(x) = x^2$ is _____
   a. Bijective
   b. Surjective but not Injective
   c. Injective but Surjective
   d. Neither Surjective nor Injective

4. Let : $N \rightarrow R$ be defined by $f(x) = x^2$. Range of the function among the following is _____
   a. {1, 4, 9, 16,…}
   b. {1, 4, 8, 9, 10,…}
   c. {1, 4, 9, 15, 16,…}
   d. {1, 4, 8, 16,…}

5. The function f: Z→Z defined by $f(x) = x^2$ is_____
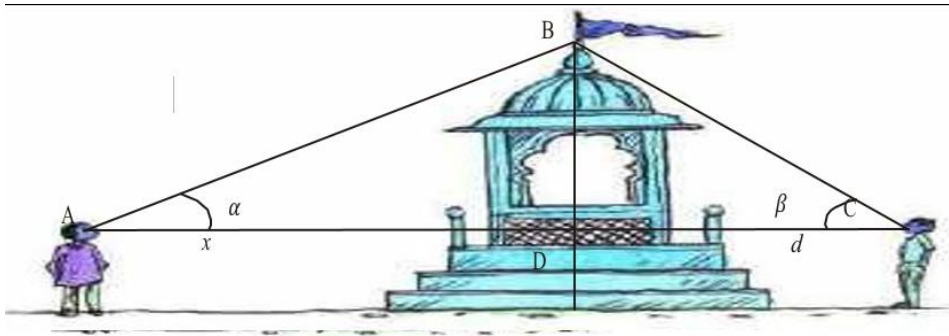   a. Neither Injective nor Surjective

b. Injective

c. Surjective

d. Bijective

ANSWERS

1. (a) Neither Surjective nor Injective
2. (C) Injective
3. (a) Bijective
4. (a) {1, 4, 9, 16,…}
5. (a) Neither Injective nor Surjective

Inverse Trigonometric Function:

CASE STUDY1:

Two men on either side of a temple of 30 meters high observe its top at the angles of elevation α and β respectively. (as shown in the figure above). The distance between the two men is $40\sqrt{3}$ meters and the distance between the first person A and the temple is $30\sqrt{3}$ meters. Based on the above information answer the following:

1. $\angle CAB = \alpha =$

   a. $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$

   b. $\sin^{-1}\left(\frac{1}{2}\right)$

   c. $\sin^{-1}(2)$

   d. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

2. $\angle CAB = \alpha =$

a. $\cos^{-1}\left(\frac{1}{5}\right)$

b. $\cos^{-1}\left(\frac{2}{5}\right)$

c. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

d. $\cos^{-1}\left(\frac{4}{5}\right)$

3. $\angle BCA = \beta =$

a. $\tan^{-1}\left(\frac{1}{2}\right)$

b. $\tan^{-1}(2)$

c. $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

d. $\tan^{-1}\left(\sqrt{3}\right)$

4. $\angle ABC =$

a. $\frac{\pi}{4}$

b. $\frac{\pi}{6}$

c. $\frac{\pi}{2}$

d. $\frac{\pi}{3}$

5. Domain and Range of $\cos^{-1} x =$

a. $(-1, 1), \ (0, \pi)$

b. $[-1, 1], \ (0, \pi)$

c. $[-1, 1], \ [0, \pi]$

d. $(-1, 1), \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

ANSWERS

1. (b) $\sin^{-1}\left(\frac{1}{2}\right)$

2. (c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

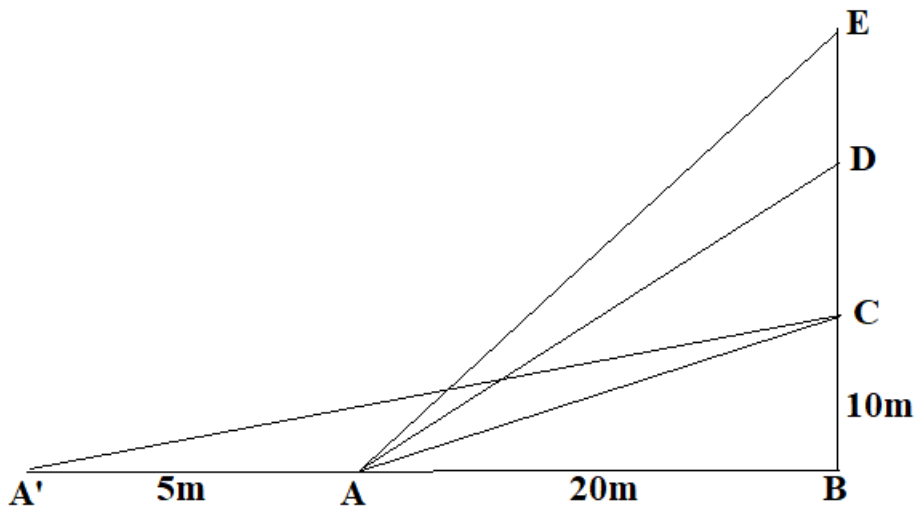**3.** (d) $\tan^{-1}\left(\sqrt{3}\right)$

4. (c) $\frac{\pi}{2}$

5. (c) $[-1, 1], [0, \pi]$

CASE STUDY 2:

The Government of India is planning to fix a hoarding board at the face of a building on the road of a busy market for awareness on COVID-19 protocol. Ram, Robert and Rahim are the three engineers who are working on this project. "A" is considered to be a person viewing the hoarding board 20 metres away from the building, standing at the edge of a pathway nearby. Ram, Robert and Rahim suggested to the firm to place the hoarding board at three different locations namely C, D and E. "C" is at the height of 10 metres from the

ground level. For the viewer A, the angle of elevation of "D" is double the angle of elevation of "C" The angle of elevation of "E" is triple the angle of elevation of "C" for the same viewer. Look at the figure given and based on the above information answer the following:

1. Measure of $\angle CAB$ =

a. $\tan^{-1}(2)$

b. $\tan^{-1}\left(\frac{1}{2}\right)$

c. $\tan^{-1}(1)$

d. $\tan^{-1}(3)$

2. *Measure of* $\angle DAB$ =

a. $\tan^{-1}\left(\frac{3}{4}\right)$

b. $\tan^{-1}(3)$

c. $\tan^{-1}\left(\frac{4}{3}\right)$

d. $\tan^{-1}(4)$

3. *Measure of* $\angle EAB$ =

a. $\tan^{-1}(11)$

b. $\tan^{-1}3$

c. $\tan^{-1}\left(\frac{2}{11}\right)$

d. $\tan^{-1}\left(\frac{11}{2}\right)$

4. A^{l} Is another viewer standing on the same line of observation across the road. If the width of the road is 5 meters, then the difference between $\angle CAB$ and $\angle CA'B$

Is

a. tan⁻¹(1/2)

b. $\tan^{-1}(1/8)$

c. $\tan^{-1}\left(\frac{2}{5}\right)$

d. $\tan^{-1}\left(\frac{11}{21}\right)$

5. Domain and Range of $\tan^{-1} x =$

a. R^+, $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

b. R^-, $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

c. R, $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

d. R, $\left(0, \frac{\pi}{2}\right)$

ANSWERS

1. (b) $\tan^{-1}\left(\frac{1}{2}\right)$

2. (c) $\tan^{-1}\left(\frac{4}{3}\right)$

3. (d) $\tan^{-1}\left(\frac{11}{2}\right)$

4. (b) $\tan^{-1}(1/8)$

5. (c) R, $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

MATRICES

CASE STUDY1:

A manufacture produces three stationery products Pencil, Eraser and Sharpener which he sells in two markets. Annual sales are indicated below

Market	Products (in numbers)		
	Pencil	Eraser	Sharpener
A	10,000	2000	18,000
B	6000	20,000	8,000

If the unit Sale price of Pencil, Eraser and Sharpener are Rs. 2.50, Rs. 1.50 and Rs. 1.00 respectively, and unit cost of the above three commodities are Rs. 2.00, Rs. 1.00 and Rs. 0.50 respectively, then,

Based on the above information answer the following:

1. Total revenue of market A
   a. Rs. 64,000
   b. Rs. 60,400
   c. Rs. 46,000
   d. Rs. 40600
2. Total revenue of market B
   a. Rs. 35,000
   b. Rs. 53,000
   c. Rs. 50,300
   d. Rs. 30,500
3. Cost incurred in market A
   a. Rs. 13,000
   b. Rs.30,100
   c. Rs. 10,300
   d. Rs. 31,000
4. Profit in market A and B respectively are
   a. (Rs. 15,000, Rs. 17,000)
   b. (Rs. 17,000, Rs. 15,000)
   c. (Rs. 51,000, Rs. 71,000)
   d. (Rs. 10,000, Rs. 20,000)
5. Gross profit in both market
   a. Rs.23,000
   b. Rs. 20,300
   c. Rs. 32,000
   d. Rs. 30,200

1. Rs. 46,000

2. Rs. 53,000

3. RS.31,000

4. (Rs.15, 000, Rs.17, 000)

5. Rs. 32,000

CASE STUDY 2:

Amit, Biraj and Chirag were given the task of creating a square matrux of order 2.

Below are the matrices created by them. A, B , C are the matrices created by Amit, Biraj and Chirag respectively.

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

If a = 4 and b = −2, based on the above information answer the following:

1. Sum of the matrices A, B and C , A+(B + C) is

   a. $\begin{bmatrix} 1 & 6 \\ 2 & 7 \end{bmatrix}$

   b. $\begin{bmatrix} 6 & 1 \\ 7 & 2 \end{bmatrix}$

   c. $\begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$

   d. $\begin{bmatrix} 2 & 1 \\ 7 & 6 \end{bmatrix}$

2. $(A^T)^T$ is equal to

   a. $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

   b. $\begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$

   c. $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

   d. $\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$

3. $(bA)^T$ is equal to

   a. $\begin{bmatrix} -2 & -4 \\ 2 & -6 \end{bmatrix}$

   b. $\begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix}$

   c. $\begin{bmatrix} -2 & 2 \\ -6 & -4 \end{bmatrix}$

d. $\begin{bmatrix} -6 & -2 \\ 2 & 4 \end{bmatrix}$

4. $AC - BC$ is equal to

a. $\begin{bmatrix} -4 & -6 \\ -4 & 4 \end{bmatrix}$

b. $\begin{bmatrix} -4 & -4 \\ 4 & -6 \end{bmatrix}$

c. $\begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$

d. $\begin{bmatrix} -6 & 4 \\ -4 & -4 \end{bmatrix}$

5. $(a + b)B$ is equal to

a. $\begin{bmatrix} 0 & 8 \\ 10 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 2 & 10 \\ 8 & 0 \end{bmatrix}$

c. $\begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$

d. $\begin{bmatrix} 2 & 0 \\ 8 & 10 \end{bmatrix}$

Answers

1. (c) $\begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$

2. (a) $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

3. (b) $\begin{bmatrix} -2 & 2 \\ -4 & -6 \end{bmatrix}$

4. (c) $\begin{bmatrix} -4 & -4 \\ -6 & 4 \end{bmatrix}$

5. (c) $\begin{bmatrix} 8 & 0 \\ 2 & 10 \end{bmatrix}$

CASE STUDY 2:

Three schools DPS, CVC and KVS decided to organize a fair for collecting money for helping the flood victims. They sold handmade fans, mats and plates from recycled material at a cost of Rs. 25, Rs.100 and Rs. 50 each respectively. The numbers of articles sold are given as

School /Article	DPS	CVC	KVS
Handmade fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Based on the information given above, answer the following questions:

1. What is the total money (in Rupees) collected by the school DPS?
a. 700
b. 7,000
c. 6;125
d. 7875

2. What is the total amount of money (in Rs.) collected by schools CVC and KVS?
a. 14,000
b. 15,725
c. 21,000
d. 13,125

3. What is the total amount of money collected by all three schools DPS, CVC and KVS?
a. Rs. 15,775
b. Rs. 14,000
c. Rs. 21,000
d. Rs. 17,125

4. If the number of handmade fans and plates are interchanged for all the schools, then what is the total money collected by all schools?
a. Rs. 18,000
b. Rs. 6,750
c. Rs. 5,000
d. Rs. 21,250

5. How many articles (in total) are sold by three schools?
a. 230
b. 130
c. 430
d. 330

ANSWERS

1. (b) 7000
2. (a) 14000
3. (c) Rs.21000
4. (d) 21250
5. (d) 330

CASE STUDY 3:

On her birth day, Seema decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got Rs.10 more. However, if there were 16 children more, everyone would have got Rs. 10 less. Let the number of children be x and the amount distributed by Seema for one child be y (in Rs.).

Based on the information given above, answer the following questions:

1. The equations in terms x and y are
a. 5x-4y = 40

5x-8y = -80

b. 5x-4y = 40

5x-8y = 80

c. 5x-4y = 40

5x+8 y= -80

d. 5x+4y = 40

5x-8y = -80

2. Which of the following matrix equations represent the information given above?

1. $\begin{bmatrix} 5 & 4 \\ 5 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$

2. $\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 80 \end{bmatrix}$

3. $\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$

4. $\begin{bmatrix} 5 & 4 \\ 5 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$

3. The number of children who were given some money by Seema, is

a. 30

b. 40

c. 23

d. 32

4. How much amount is given to each child by Seema?

a. Rs. 32

b. Rs. 30

c. Rs. 62

d. Rs. 26

5. How much amount Seema spends in distributing the money to all the students of the Orphanage?

a. Rs. 609

b. Rs. 960

c. Rs. 906

d. Rs. 690

ANSWERS

1. (a) 5x-4y = 40

5x-8y = -80

2. (c) $\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$

3. (d) 32

4. (b) Rs.30

5. (b) Rs.960

CASE STUDY 4:

Two farmers Ramakishan and Gurucharan Singh cultivate only three varieties of rice namely Basmati, Permal and Naura. The sale (in rupees) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B

September sales (in Rupees)

$$A = \begin{bmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{bmatrix} \begin{matrix} Ramakishan \\ Gurucharan \end{matrix}$$

October sales (in Rupees)

$$B = \begin{bmatrix} 5,000 & 10,000 & 6,000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix} \begin{matrix} Ramakishan \\ Gurucharan \end{matrix}$$

1. The total sales in September and October for each farmer in each variety can be represented as _____..

a. A+B

b. A-B

c. A$> B$

d. A$< B$

2. What is the value of A_{23}?

a. 10000

b. 20000

c. 30000

d. 40000

3. The decrease in sales from September to October is given by _____ .

a. A+B

b. A-B

c. A$> B$

d. A$< B$

4. If Ramkishan receives 2% profit on gross sales, compute his profit for each variety sold in October.

a. Rs. 100, Rs. 200 and Rs. 120

b. Rs. 100, Rs. 200 and Rs. 130

c. Rs. 100, Rs. 220 and Rs. 120

d. Rs. 110, Rs. 200 and Rs. 120

5. If Gurucharan receives 2% profit on gross sales, compute his profit for each variety sold in September.

a. Rs. 100, Rs. 200, Rs. 120

b. Rs. 1000 , Rs. 600, Rs. 200

c. Rs. 400, Rs. 200, Rs. 120

d. Rs. 1200, Rs. 200, Rs. 120

ANSWERS

1. (a) A+B

2. (a) 10000

3. (b) A-B

4. (a) Rs. 100, Rs. 200 and Rs. 120

5. (b) Rs. 1000, Rs. 600, Rs. 200

Determinants

CASE STUDY 1:

Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50m, then its area will remain same, but if length is decreased by 10m and breadth is decreased by 20m, then its area will decrease by 5300 m^2
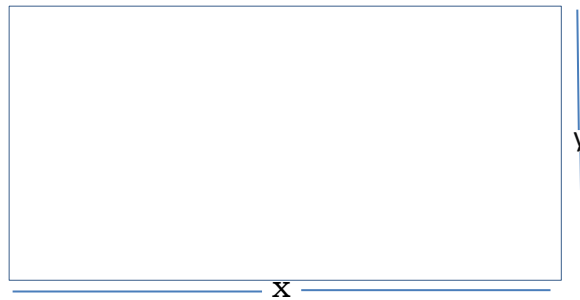
Based on the information given above, answer the following questions:

1. The equations in terms of X and Y are

a. x-y=50, 2x-y=550

b. x-y=50, 2x+y=550

c. x + y = 50, 2x + y=550

d. x +y = 50, 2x + y=550

2. Which of the following matrix equation is represented by the given   information

a. $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -50 \\ -550 \end{bmatrix}$

3. The value of x (length of rectangular field) is

a. 150m

b. 400m

c. 200m

d. 320m

4. The value of y (breadth of rectangular field) is

a. 150m.

b. 200m.

c. 430m.

d. 350m

5. How much is the area of rectangular field?

a. 60000Sq.m.

b. 30000Sq.m.

c. 30000m

d. 3000m

<h2 style="text-align:center;">ANSWERS</h2>

1. b) x-y=50, 2x+y=550

2. a) $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

3. c) 200m

4. a) 150m

5. b) 30000Sq.m

<h2 style="text-align:center;">Continuity and Differentiability</h2>

CASE STUDY 1:

The Relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the following equation $y = 4x - \frac{1}{2}x^2$ where x is the number of days exposed to sunlight.

1. The rate of growth of the plant with respect to sunlight is _____ .

a. $4x - \frac{1}{2}x^2$

b. $4 - x$

c. $x - 4$

d. $x - \frac{1}{2}x^2$

2. What is the number of days it will take for the plant to grow to the maximum height?

a. 4

b. 6

c. 7

d. 10

3. What is the maximum height of the plant?

a. 12 cm

b. 10 cm

c. 8 cm

d. 6 cm

4. What will be the height of the plant after 2 days?

a. 4cm

b. 6 cm

c. 8cm

d. 10cm

5. If the height of the plant is 7/2 cm, the number of days it has been exposed to the sunlight is _____ .

a. 2

b. 3

c. 4

d. 1

ANSWERS

1. b) 4 – x

2. a) 4

3. c) 8cm

4. b) 6 cm

5. d) 1

CASE STUDY 2:

$P(x) = -5x^2 + 125x + 37500$ is the total profit function of a company, where x is the production of the company.

1. What will be the production when the profit is maximum?

a. 37500

b. 12.5

c. -12.5

d. –37500

2. What will be the maximum profit?

a. Rs 38,28,125

b. Rs 38281.25

c. Rs 39,000

d. None

3. Check in which interval the profit is strictly increasing .

a. $(12.5, \infty)$

b. for all real numbers

c. for all positive real numbers

d. $(0, 12.5)$

4. When the production is 2units what will be the profit of the company?

a. 37500

b. 37,730

c. 37,770

d. None

5. What will be production of the company when the profit is Rs 38250?

a. 15

b. 30

c. 2

d. data is not sufficient to find

1. b) 12.5
2. b) Rs.38281.25
3. d) (0 , 12.5)
4. b) 37,730
5. a) 15

CASE STUDY 3:

A potter made a mud vessel, where the shape of the pot is based on   f(x) = |x – 3| + | x-2|, where f(x) represents the height of the pot.

1. When x > 4 What will be the height in terms of x ?
a. x – 2
b.   x -3
c. 2x -5
d. 5 – 2x

2. Will the slope vary with x value?
a. Yes
b. No

3. What is $\frac{dy}{dx}$ at x = 3
a. 2
b. -2
c. Function is not differentiable
d. 1

4. When the x value lies between (2,3) then the function is
a. 2x -5
b. 5 – 2x
c. 1

d. 5

5. If the potter is trying to make a pot using the function f(x) = [x] ,will he get a pot or not? Why?

a. Yes, because it is a continuous function

b. Yes, because it is not continuous

c. No , because it is a continuous function

d. No , because it is not continuous

ANSWERS

1. c) 2x -5
2. a) yes
3. c) function is not differentiable
4. c) 1
5. d) No, because it is not continuous

CASE STUDY4:

The shape of a toy is given as f(x) = 6(2x⁴ – x²). To make the toy beautiful 2 sticks which are perpendicular to each other were placed at a point (2,3), above the toy.

1. Which value from the following may be abscissa of critical point?

a. $\pm\frac{1}{4}$

b. $\pm\frac{1}{2}$

c. ± 1

d. None

2. Find the slope of the normal based on the position of the stick.

a. 360

b. –360

c. $\dfrac{1}{360}$

d. $\dfrac{-1}{360}$

3. What will be the equation of the tangent at the critical point if it passes through (2,3)?

a. x + 360y = 1082

b. y = 360x – 717

c. x = 717y + 360

d. none

4. Find the second order derivative of the function at x = 5.

a. 598

b. 1176

c. 3588

d. 3312

5. At which of the following intervals will f(x) be increasing?

a. (-∞, -1/2) U (1/2, ∞)

b. (-1/2, 0) U (1/2, ∞)

c. (0, ½) U (1/2, ∞)

d. (-∞, -1/2) U (0, ½)

ANSWERS

1. b) $\pm\dfrac{1}{2}$

2. d) $\dfrac{-1}{360}$

3. b) y = 360x – 717

4. c) 3588

5. b) (-1/2, 0) U (1/2, ∞)

CASE STUDY 5:
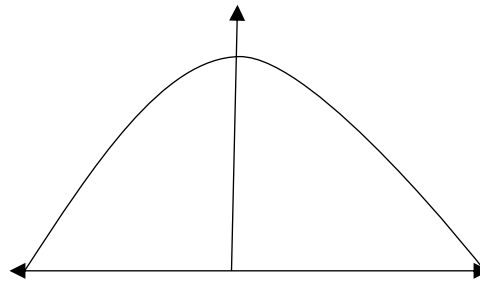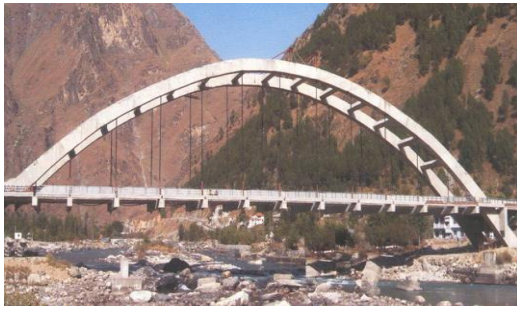
The bridge connects two hills 100 feet apart. The arch on the bridge is in a parabolic form. The highest point on the bridge is 10 feet above the road at the middle of the bridge as seen in the figure.

Based on the information given above, answer the following questions:

1. The equation of the parabola designed on the bridge is
   a. $x^2 = 250y$
   b. $x^2 = -250y$
   c. $y^2 = 250x$
   d. $y^2 = 250y$

2. The value of the integral $\int_{-50}^{50} \frac{x^2}{250} dx$ is

   a. $\dfrac{1000}{3}$

   b. $\dfrac{250}{3}$

   c. 1200
   d. 0

3. The integrand of the integral $\int\limits_{-50}^{50} x^2 dx$ is _____ function.

   a. Even
   b. Odd
   c. Neither odd nor even
   d. None

4. The area formed by the curve $x^2 = 250y$, x-axis , $y = 0$ and $y = 10$ is

   a. $\dfrac{1000\sqrt{2}}{3}$

b. $\dfrac{4}{3}$

c. $\dfrac{1000}{3}$

d. 0

5. The area formed between $x^2 = 250y$, y-axis , $y = 2$ and $y = 4$ is

a. $\dfrac{1000}{3}$

b. 0

c. $\dfrac{1000\sqrt{2}}{3}$

d. none of these

ANSWERS

1. b) $x^2 = -250y$

2. a) $\dfrac{1000}{3}$

3. a) Even

4. c) $\dfrac{1000}{3}$

5. d) none of these

Differential Equation

CASE STUDY 1:

A Veterinary doctor was examining a sick cat brought by a pet lover. When it was brought to the hospital, it was already dead. The pet lover wanted to find its time of death. He took the temperature of the cat at 11.30 pm which was 94.6°F. He took the temperature again after one hour; the temperature was lower than the first observation. It was 93.4°F. The room in which the cat was put is always at 70°F. The normal temperature of the cat is taken as 98.6°F when it was alive. The doctor estimated the time of death using *Newton law of cooling which is governed by the differential equation:* $\dfrac{dT}{dt} \propto (T - 70)$, where 70°F is the room temperature and T is the temperature *of the object* at time *t.*

Substituting the two different observations of T and t made, in the solution of the differential equation $\dfrac{dT}{dt} = k(T - 70)$ where k is a constant of proportion, time of death is calculated.

1. State the degree of the above given differential equation.
2. Which method of solving a differential equation helped in calculation of the time of death?
   a. Variable separable method
   b. Solving Homogeneous differential equation
   c. Solving Linear differential equation
   d. all of the above
3. If the temperature was measured 2 hours after 11.30pm, will the time of death change? (Yes/No)
4. The solution of the differential equation $\frac{dT}{dt} = k(T - 70)$ is given by,
   a. log | T – 70| = kt + C
   b. log | T – 70| = log |kt |+ C
   c. T – 70 = kt + C
   d. T – 70 = kt C

5. If t = 0 when T is 72, then the value of c is
   a. -2
   b. 0
   c. 2
   d. Log 2

ANSWERS

1. Degree is 1
2. (a) Variable separable method
3. No
4. (a) log | T – 70| = kt + C
5. (d) log 2

CASE STUDY 2:

Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $\frac{dy}{dx} = k(50 - y)$ where x denotes the number of weeks and y the number of children who have been given the drops.

1.State the order of the above given differential equation.

2.Which method of solving a differential equation can be used to solve $\frac{dy}{dx} = k(50 - y).?$

   a. Variable separable method

   b. Solving Homogeneous differential equation

   c. Solving Linear differential equation

   d. all of the above

3. The solution of the differential equation $\frac{dy}{dx} = k(50 - y)$ is given by,

   a. log | 50 – y| = kx + C

   b. - log | 50 – y| = kx + C

   c. log | 50 – y| = log| kx |+ C

   d. 50 – y = kx + C

4. The value of c in the particular solution given that y(0)=0 and k = 0.049 is.

   a. log 50

   b. log 1/50

   c. 50

   d. -50

5. Which of the following solutions may be used to find the number of children who have been given the polio drops?

   a. $y = 50 - e^{kx}$

   b. $y = 50 - e^{=kx}$

   c. $y = 50 (1 - e^{-kx})$

   d. $y = 50 (e^{=kx} - 1)$

ANSWERS:

   1. Order is 1

   2. (a) Variable separable method

   3. (b) - log | 50 – y| = kx + C

   4. (b) log 1/50

   5. (c) $y = 50 (1 - e^{-kx})$

Vector Algebra

CASE STUDY1:

Solar Panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels.

A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture , suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of meters P_1 (6,8,4) , P_2 (21,8,4), P_3 (21,16,10) and

P_4 (6,16,10)

1. What are the components to the two edge vectors defined by $\vec{A}$ = PV of P_2 – PV of P_1 and $\vec{B}$ = PV of P_4 – PV of P_1? (where PV stands for position vector)
2. Write the vector in standard notation with $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ (where $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are the unit vectors along the three axes).
3. What are the magnitudes of the vectors $\vec{A}$ and $\vec{B}$ and in what units?
4. What are the components to the vector $\vec{N}$, perpendicular to $\vec{A}$ and $\vec{B}$ and the surface of the roof?
5. What is the magnitude of $\vec{N}$ and its units? The sun is located along the unit vector $\vec{S}$ = ½ $\hat{\imath}$-6/7$\hat{\jmath}$ +1/7 $\hat{k}$. If the flow of solar energy is given by the vector $\vec{F}$ = 910 S in units of watts/meter^2, what is the dot product of vectors $\vec{F}$ with $\vec{N}$, and the units for this quantity?
6. What is the angle between vectors $\vec{N}$ and $\vec{S}$? What is the elevation angle of the sun above the plane of the roof? (COS 51^0 =0.629)

ANSWERS

1. 15, 0, 0 : 0, 8, 6

2. Answer 15$\boldsymbol{i}$+0$\boldsymbol{j}$+0$\boldsymbol{k}$ Answer 2: 0i+8$\boldsymbol{j}$+6$\boldsymbol{k}$

3. Answer : 15 unit , Answer : $\sqrt{8^2 + 6^2} = \sqrt{64 + 36}$=√100 = 10 unit

4. $\vec{N} = \vec{A} \times \vec{B}$

$$N = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 15 & 0 & 0 \\ 0 & 8 & 6 \end{vmatrix} = \text{-15}(6\boldsymbol{j}\text{-8}\boldsymbol{k}) = \text{-90}\boldsymbol{j}\text{+120}\boldsymbol{k}$$; Answer -90 , 120

5. $\sqrt{(-90)^2 + 120^2}$=$\sqrt{8100 + 14400}$ =$\sqrt{22500}$ = 150

Answer of second part: $\vec{F}$ = 910 (1/2$\hat{\imath}$ -6/7$\hat{\jmath}$ +1/7$\widehat{\boldsymbol{k}}$) = 455$\hat{\imath}$ − 780$\hat{\jmath}$ + 130$\widehat{\boldsymbol{k}}$.

The dot product is just $\vec{F} \cdot \vec{N}$ = 455*(0) -780*(-90) + 130*120 = **85,800 watts**.

From the definition of dot product: $\vec{F} \cdot \vec{N} = |\vec{F}||\vec{N}|\cos\theta$

Then since $| \vec{F}| = 910$ and $|\vec{N}| = 150$ and $\vec{F} \cdot \vec{N} = 85,800$ we have $\cos\theta = (85800/(910 \times 150)) = 0.629$ and so $\theta =\cos^{-1}(0.629)$ **which is 0.8905 rad and is 51°**.(*using cosine table*)

This is the angle between the normal to the surface and the incident solar rays.

The compliment of this is the elevation of the sun above the plane of the roof or 90-51 = 39°.

CASE STUDY 2:

A class XII student appearing for a competitive examination was asked to attempt the following questions.

Let $\vec{a}, \vec{b},$ and $\vec{c}$ *be three* non zero vectors.

1. If $\vec{a}$ and $\vec{b}$ are such that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then

a. $\vec{a} \perp \vec{b}$

b. $\vec{a} \parallel \vec{b}$

c. $\vec{a} = \vec{b}$

d. None of these

2. If $\vec{a} = \hat{\imath} - 2\hat{\jmath} , \quad \vec{b} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ then evaluate $(2\vec{a} + \vec{b}) \cdot [(\vec{a} + \vec{b}) \times (\vec{a} - 2\vec{b})]$

a. 0

b. 4

c. 3

d. 2

3. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ be the angle between them then $|\vec{a} - \vec{b}|$ is

a. $sin\dfrac{\theta}{2}$

b. $2 sin\dfrac{\theta}{2}$

c. $2 cos\dfrac{\theta}{2}$

d. $cos\dfrac{\theta}{2}$

4. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\dfrac{\pi}{6}$ then $\vec{a} =$

a. $2(\vec{b} \times \vec{c})$

b. $-2(\vec{b} \times \vec{c})$

c. $\pm 2(\vec{b} \times \vec{c})$

d. $2(\vec{b} \pm \vec{c})$

5. The area of the parallelogram formed by $\vec{a}$ and $\vec{b}$ as diagonals is

a. 70

b. 35

c. √70/2

d. √70

ANSWERS

1. (a) $\left|\vec{a} + \vec{b}\right|^2 = \left|\vec{a} - \vec{b}\right|^2 \Rightarrow 2.\vec{a} \cdot \vec{b} = 0, \vec{a} \perp \vec{b}$

2. (a) 0

3. (b) $2 \sin\frac{\theta}{2}$

4. (c) $\pm 2(\vec{b} \times \vec{c})$

5. (c) √70/2 sq units

CASE STUDY 3:

A cricket match is organized between two Clubs A and B for which a team from each club is chosen. Remaining players of Club A and Club B are respectively sitting on the plane represented by the equation $\vec{r} \cdot (2\vec{\imath} - \vec{\jmath} + \vec{k}) = 3$ and $\vec{r} \cdot (\vec{\imath} + 3\vec{\jmath} + 2\vec{k}) = 8$, to cheer the team of their own clubs.

Based on the above answer the following:

1. The Cartesian equation of the plane on which players of Club A are seated is

a. $2x - y + z = 3$

b. $2x - y + 2z = 3$

c. $2x - y + z = -3$

d. $x - y + z = 3$

2. The magnitude of the normal to the plane on which players of club B are seated, is

a. $\sqrt{15}$

b. $\sqrt{14}$

c. $\sqrt{17}$

d. $\sqrt{20}$

3. The intercept form of the equation of the plane on which players of Club B are seated is

a. $\frac{x}{8} + \frac{y}{\frac{8}{3}} + \frac{z}{2} = 1$

b. $\frac{x}{5} + \frac{y}{\frac{8}{3}} + \frac{z}{3} = 1$

c. $\frac{x}{8} + \frac{y}{\frac{8}{3}} + \frac{z}{4} = 1$

d. $\frac{x}{8} + \frac{y}{7} + \frac{z}{2} = 1$

4. Which of the following is a player of Club B?

a. Player sitting at (1, 2, 1)

b. Player sitting at (0, 1, 2)

c. Player sitting at (1, 4, 1)

d. Player sitting at (1, 1, 2)

5. The distance of the plane, on which players of Club B are seated, from the origin is

a. $\frac{8}{\sqrt{14}}$ $units$

b. $\frac{6}{\sqrt{14}}$ $units$

c. $\frac{7}{\sqrt{14}}$ $units$

d. $\frac{9}{\sqrt{14}}$ $units$

ANSWERS

1. (a) $2x - y + z = 3$

2. (b) $\sqrt{14}$_

3. (c) $\frac{x}{8} + \frac{y}{\frac{8}{3}} + \frac{z}{4} = 1$

4. (d) Player sitting at (1, 1, 2)

5. (a) $\frac{8}{\sqrt{14}}$ $units$

CASE STUDY 2:

The Indian coast guard, while patrolling, saw a suspicious boat with people. They were nowhere looking like fishermen. The coast guard were closely observing the movement of the boat for an opportunity to seize the boat. They observed that the boat is moving along a planar surface. At an instant of time, the coordinates of the position of the coast guard helicopter and the boat is (1, 3, 5) and (2, 5, 3) respectively.

Based on the above answer the following:

1. If the line joining the positions of the helicopter and the boat is perpendicular to the plane in which the boat moves, then the equation of the plane is
   a. -x + 2y – 2z = 6
   b. x + 2y + 2z = 6
   c. x + 2y – 2z = 6
   d. x - 2y – 2z = 6

2. If the coast guard decide to shoot the boat at that given instant of time, then what is the distance (in meters) that the bullet has to travel?
   a. 5m
   b. 3m
   c. 6m
   d. 4m

3. If the coast guard decides to shoot the boat at that given instant of time, when the speed of bullet is 36m/sec, then what is the time taken for the bullet to travel and hit the boat?
   a. $\frac{1}{8} seconds$
   b. $\frac{1}{14} seconds$

c. $\frac{1}{10}$ seconds

d. $\frac{1}{12}$ seconds

4. At that given instant of time, the equation of line passing through the positions of the helicopter and boat is

a. $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z-5}{-2}$

b. $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z-5}{-2}$

c. $\frac{x+1}{-2} = \frac{y-3}{-1} = \frac{z-5}{-2}$

d. $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z+5}{2}$

5. At a different instant of time, the boat moves to a different position along the planar surface. What should be the coordinates of the location of the boat if the coast guard shoots the bullet along the line whose equation is $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{1}$ for the bullet to hit the boat?

a. $(\frac{-8}{3}, \frac{19}{3}, \frac{-14}{3})$

b. $(\frac{8}{3}, \frac{-19}{3}, \frac{-14}{3})$

c. $(\frac{8}{3}, \frac{-19}{3}, \frac{14}{3})$

d. none of the above

ANSWERS

1. (c) x + 2y – 2z = 6
2. (b) 3m
3. (d) $\frac{1}{12}$ seconds
4. (a) $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z-5}{-2}$
5. (d)_ None of the above

CASE STUDY 3:

The equation of motion of a missile are x = 3t, y = -4t, z = t, where the time 't' is given in seconds, and the distance is measured in kilometres.

Based on the above answer the following:

1. What is the path of the missile?
a. Straight line
b. Parabola
c. Circle
d. Ellipse
2. Which of the following points lie on the path of the missile?
a. (6, 8, 2)
b. (6, -8, -2)
c. (6, -8, 2)
d. (-6, -8, 2)
3. At what distance will the rocket be from the starting point (0, 0, 0) in 5 seconds?
a. $\sqrt{550}$ kms
b. $\sqrt{650}$ kms
c. $\sqrt{450}$ kms
d. $\sqrt{750}$ kms
4. If the position of rocket at a certain instant of time is (5, -8, 10), then what will be the height of the rocket from the ground? (The ground is considered as the xy – plane).
a. 12 km
b. 11 km
c. 20 km
d. 10 km
5. At a certain instant of time, if the missile is above the sea level, where the equation of the surface of sea is given by 2x + y + 3z = 1 and the position of the missile at

that instant of time is (1, 1, 2), then the image of the position of the rocket in the sea is

a. $(\frac{-9}{7}, \frac{-1}{7}, \frac{-10}{7})$

b. $(\frac{9}{7}, \frac{-1}{7}, \frac{-10}{7})$

c. $(\frac{-9}{7}, \frac{1}{7}, \frac{-10}{7})$

d. $(\frac{-9}{7}, \frac{-1}{7}, \frac{10}{7})$

ANSWERS

1. (a) Straight line
2. (c) (6, -8, 2
3. (b) $\sqrt{650}$ kms
4. (d) 10 km
5. (a) $(\frac{-9}{7}, \frac{-1}{7}, \frac{-10}{7})$

CASE STUDY 4:

Suppose the floor of a hotel is made up of mirror polished Salvatore stone. There is a large crystal chandelier attached to the ceiling of the hotel room. Consider the floor of the hotel room as a plane having the equation x – y + z = 4 and the crystal chandelier is suspended at the point (1, 0, 1).

Based on the above answer the following:

1. Find the direction ratios of the perpendicular from the point (1, 0, 1) to the plane

    x – y + z = 4.

a. (-1, -1, 1)

b. (1, -1, -1)

c. (-1, -1, -1)

d. (1, -1, 1)

2. Find the length of the perpendicular from the point (1, 0, 1) to the plane x – y + z = 4.

a. $\frac{2}{\sqrt{3}}$ units

b. $\frac{4}{\sqrt{3}}$ units

c. $\frac{6}{\sqrt{3}}$ units

d. $\frac{8}{\sqrt{3}}$ units

3. The equation of the perpendicular from the point (1, 0, 1) to the plane x – y + z = 4 is

a. $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z+5}{2}$

b. $\frac{x-1}{-2} = \frac{y+3}{-1} = \frac{z-5}{2}$

c. $\frac{x-1}{1} = \frac{y}{-1} = \frac{z-1}{1}$

d. $\frac{x-1}{2} = \frac{y}{-2} = \frac{z-1}{1}$

4. The equation of the plane parallel to the plane x – y + z = 4, which is at a unit distance from the point (1, 0, 1) is

a. x – y + z + (2 - $\sqrt{3}$)

b. x – y + z - (2 + $\sqrt{3}$)

c. x – y + z + (2 + $\sqrt{3}$)

d. Both (a) and (c)

5. The direction cosine of the normal to the plane x – y + z = 4 is

a. $(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}})$

b. $(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

c. $\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

d. $(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}})$

ANSWERS

1. (d) (1, -1, 1)

2. (a) $\frac{2}{\sqrt{3}}$ units

3. (c) $\frac{x-1}{1} = \frac{y}{-1} = \frac{z-1}{1}$

4. (d) Both (a) and (c)

5. (b) $(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

CASE STUDY 5:

A mobile tower stands at the top of a hill. Consider the surface on which the tower stands as a plane having points A(1, 0, 2), B(3, -1, 1) and C(1, 2, 1) on it. The mobile tower is tied with 3 cables from the point A, B and C such that it stands vertically on the ground. The top of the tower is at the point (2, 3, 1) as shown in the figure.
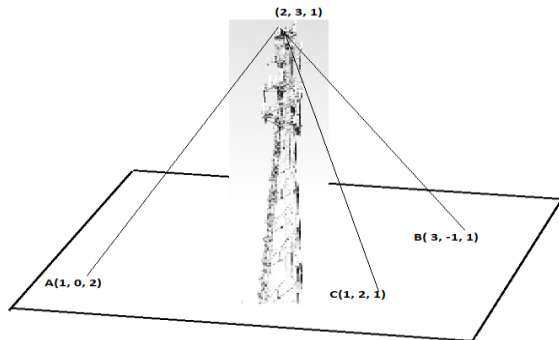
Based on the above answer the following:

1. The equation of the plane passing through the points A, B and C is
a. 3x - 2y + 4z = -11
b. 3x + 2y + 4z = 11
c. 3x - 2y - 4z = 11
d. -3x + 2y + 4z = -11

2. The height of the tower from the ground is

a. $\frac{5}{\sqrt{29}}$ units

b. $\frac{7}{\sqrt{29}}$ units

c. $\frac{6}{\sqrt{29}}$ units

d. $\frac{8}{\sqrt{29}}$ units

3. The equation of the perpendicular line drawn from the top of the tower to the ground is

a. $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z-5}{-2}$

b. $\frac{x-2}{-3} = \frac{y-3}{2} = \frac{z-1}{-4}$

c. $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-1}{4}$

d. $\frac{x+1}{-2} = \frac{y+3}{-1} = \frac{z-5}{2}$

4. The coordinates of the foot of the perpendicular drawn from the top of the tower to the ground are

a. $(\frac{43}{29}, \frac{-77}{29}, \frac{-9}{29})$

b. $(\frac{9}{7}, \frac{-1}{7}, \frac{-10}{7})$

c. $(\frac{-43}{29}, \frac{77}{29}, \frac{-9}{29})$

d. $(\frac{43}{29}, \frac{77}{29}, \frac{9}{29})$

5. The area of $\Delta\ ABC$ is

a. $\frac{\sqrt{29}}{4}$ sq. units

b. $\frac{\sqrt{29}}{2}$ sq. units

c. $\frac{\sqrt{39}}{2}$ sq. units

d. $\frac{\sqrt{39}}{4}$ sq. units

ANSWERS

1. (b) 3x + 2y + 4z = 11

2. (a) $\frac{5}{\sqrt{29}}\ units$

3. (c)) $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-1}{4}$

4. (d) $(\frac{43}{29}, \frac{77}{29}, \frac{9}{29})$

5. (b)) $\frac{\sqrt{29}}{2}$ sq. units

Probability

CASE STUDY 1:

A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots

From this situation answer the following:

1. Let the target is hit by A, B: the target is hit by B and, C: the target is hit by A and C. Then, the probability that A, B and, C all will hit, is
   a. 4/5
   b. 3/5
   c. 2/5
   d. 1/5

2. Referring to (i), what is the probability that B, C will hit and A will lose?
   a. 1/10
   b. 3/10
   c. 7/10
   d. 4/10

3. With reference to the events mentioned in (i), what is the probability that 'any two of A, B and C will hit?
   1. 1/30
   2. 11/30
   3. 17/30
   4. 13/30

4. What is the probability that 'none of them will hit the target'?
   a. 1/30

b. 1/60

c. 1/15

d. 2/15

5. What is the probability that at least one of A, B or C will hit the target?

a. 59/60

b. 2/5

c. 3/5

d. 1/60

Answers:

1. (c) 2/5

2. (a) 1/10

3. (d) 13/30

4. (b) 1/60

5. (a) 59/60

CASE STUDY 2:

The reliability of a COVID PCR test is specified as follows:

Of people having COVID, 90% of the test detects the disease but 10% goes undetected. Of people free of COVID, 99% of the test is judged COVID negative but 1% are diagnosed as showing COVID positive. From a large population of which only 0.1% have COVID, one person is selected at random, given the COVID PCR test, and the pathologist reports him/her as COVID positive.

 Based on the above information, answer the following

1. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually having COVID?
   a. 0.001
   b. 0.1
   c. 0.8
   d. 0.9

2. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually not having COVID'?
   a. 0.01
   b. 0.99
   c. 0.1
   d. 0.001

3. What is the probability that the 'person is actually not having COVID?
   a. 0.998
   b. 0.999
   c. 0.001
   d. 0.111

4. What is the probability that the 'person is actually having COVID given that 'he is tested as COVID positive'?
   a. 0.83
   b. 0.0803
   c. 0.083
   d. 0.089

5. What is the probability that the 'person selected will be diagnosed as COVID positive'?
   a. 0.1089
   b. 0.01089
   c. 0.0189
   d. 0.189

Answers

1. (d) 0.9
2. (a) 0.01
3. (b) 0.999
4. (c) 0.083

5. (b) 0.01089

CASE STUDY 3:

In answering a question on a multiple choice test for class XII, a student either knows the answer or guesses. Let 3/5 be the probability that he knows the answer and 2/5 be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability 1/3. Let E_1, E_2, E be the events that the student knows the answer, guesses the answer and answers correctly respectively.

Based on the above information, answer the following

1. What is the value of $P(E_1)$?

a. 2/5

b. 1/3

c. 1

d. 3/5

2. Value of $P(E \mid E_1)$ is

a. 1/3

b. 1

c. 2/3

d. 415

3. $\sum_{k=1}^{k=2} P(E|E_k) P(E_k)$ Equals

a. 11/15

b. 4/15

c. 1/5

d. 1

4. Value of $\sum_{k=1}^{k=2} P(E_k)$

a. 1/3

b. 1/5

c. 1

d. 3/5

5. What is the probability that the student knows the answer given that he answered it correctly?

a. 2/11

b. 5/3

c. 9/11

d. 13/3

Answers

1. (d) 3/5
2. (b) 1
3. (a) 11/15
4. (c) 1
5. (c) 9/11